**Barron’s Let’s Review Regents – Algebra II**

# Chapter 6: Systems of Equations

## 6.1 Two Equations with Two Unknowns

**Key Ideas**

A *system of two equations* with two unknowns is two equations each containing two variables. The solution set to a system of two equations with two unknows is usually an ordered pair that makes both equations true.

The ordered pair (3, 7) is a solution to the equation . If the first number for the and if the second number is substituted for the , the equation becomes , which is true. Other ordered pairs that are solutions to this equation are (8, 2), (5, 5), (10, 0), and (12, -2).

An example of a system of two equations with two unknowns is

The goal is to find, if one exists, and ordered pair that satisfies both equations at the same time. By examining different ordered pairs that make the first equation true and by testing to see if they also make the second equation true, it is possible through the process of guess-and-check to find that the ordered pair (7, 3) is the solution because and   
. For more complicated examples, though, a method that uses algebra is needed.

**The Substitution Method**

If one of the variables is already isolated in one of the equations, it is possible to simplify the two equations with two variables into just one equation with one variable.

Since is isolated in the first equation, the in the second equation can be replaced with .

To get the other number in the ordered pair, substitute into the first equation.

The ordered pair that satisfies both equations is (4, 10).

**Example 1**

Find the solution set of the system of equations.

*Solution*: Substitute for in the second equation.

Substitute into one of the original equations.

The solution is (6, 7).

**The Elimination Method**

If the two equations can be combined in a way that eliminates one of the variables, the remaining equation can be solved for one part of the ordered pair. Sometimes, this method requires that you multiply both sides of one or of both equations by some constant.

**Case 1:**

A coefficient of one variable is the opposite of the coefficient of the same variable in the other equation.

Examine the coefficients of the system of equations.

The coefficient of the in the first equation is 3.

The coefficient of the in the second equation is -3. Rember that . If these equations are added together, the result with be an equation with no -term.

Substitute into either original equation and solve for .

The solution to the system of equations is the ordered pair (4, 1).

**Case 2:**

A coefficient of one of the variables in one of the equations is equal to the coefficient of the same variable in the other equation.

Examine the coefficients in the system of equations.

The coefficients of the -terms in both equations are equal to +4.

It is permitted to multiply both sides of an equation by the same constant. For this situation, multiply both sides of either equation by -1. Below is what happens if both sides of the second equation are multiplied by -1.

Substitute 3 for in either of the original equations.

The solution set is (5, 3).

**Case 3:**

A coefficient of one of the variables in one of the equations is a multiple of the same variable in the other equation.

Examine the coefficients in the system of equations.

The coefficient of the in the first equation, 6, is a multiple of the coefficient of the in the second equation, 3. For the equation that has the smaller coefficient, multiply both sides of the equation by the number that would make the coefficient the opposite of the one from the other equation. For this example, multiply both sides of the second equation by -2.

Now substitute into one of the original equations.

The solution is (8, -2).

**Case 4:**

No coefficient for either variable in either equation is a multiple of the coefficient of the same variable in the other equation.

Examine the coefficients of the system of equations.

For the -terms, the 5 is not a multiple of 3. For the   
-terms, the -6 is not a multiple of the 4. When this happens, both equations must be changed.

You can choose whether you want to change the equations so that either the -variables will be eliminated or the -variables will be eliminated.

To eliminate the -variables, find the least common multiple of the two coefficients. In this case, the least common multiple of 4 and 6 is 12. The goal is to convert one of the y-coefficients into a +12 and the other into a -12. This can be done by multiplying both sides of the first equation by +3 and both sides of the second equation by +2.

Substitute 10 for into either of the original equations.

The solution is (10, 3).

**Example 2**

Solve the system of equations by eliminating the -terms.

Solution: The least common multiple of the   
-coefficients is 12. Multiply both sides of the first equation by +4 and both sides of the second equation by -3.

Substitute into one of the original equations.

The solution is (6, -2).

**Math Facts**

Sometimes when solving a system of equations with the elimination, both variables get eliminated and the resulting equation has just numbers. If the two numbers of the resulting equation are not equal, there is no solution to the original system. If the two numbers of the resulting equation are equal, there are an infinite number of solutions to the equation.

**Example 3**

How many solutions does this system of equations have?

(1) No solutions  
(2) One solution  
(3) Two solutions  
(4) More than two solutions

Solution: When the two equations are added together, the result is , which is never true. There are no solutions to the original system of equations. Choice (1) is correct.

**Linear-Quadratic Systems of Equations**

If one of the equations has an exponent of 2, the system can be solved with the substitution method. The resulting equation will be a quadratic equation with up to two solutions.

**Example 4**

What ordered pair(s) satisfies the system?

*Solution*:

For . For . So the two solutions are (-1, 0) and (2, 3).

### Check Your Understanding of Section 6.1

1. *Multiple-Choice*
2. Solve the system of equations:  
   **(4) (1, 3)**
3. Solve the system of equations.  
   **(1) (3, 7)**
4. Solve the system of equations.  
   **(1) (9, 2)**
5. Solve the system of equations.  
   **(3) (-5, 3)**
6. Solve the system of equations.  
   **(1) (4, 1)**
7. Solve the system of equations.  
   **(2) (5, 2)**
8. Solve the system of equations.  
   **(1) (3, -4)**
9. Solve the system of equations.  
   **(4) No solution**
10. How many solutions does the system of equations have?  
    **(4) Infinite solutions**
11. If a system of two equations with two unknowns has no solution, what do the graphs of the two lines representing the two equations look like?  
    **(2) They are parallel**
12. *Show how you arrived at your answers*.
13. When will it be easier to use the substitution method rather than the elimination method when solving a system of two equations with two unknowns.  
      
    The substitution method is easier if one of the variables is already isolated in one of the equations, it is possible to simplify the two equations with two variables into just one equation with one variable.
14. Reginald is solving the system of equations.   
    Reginald adds the two equations to get . Madelyn subtracts the two equations to get . Who is correct?  
      
    Both are correct. Reginald first gets and then by substation: .  
      
    Madelyn first gets . Then by substation: .  
      
    Both get the same solution: (10, 2).
15. Solve the system of equations.  
      
    Solution: (-4, 2)
16. Movie tickets costs $7 for children and $12 for adults. If 10 people purchase tickets and the total is $85, how many adult tickets and how many children tickets were purchased?  
     is the number of children tickets purchased  
     is the number of adult tickets purchased  
      
    7 children tickets and 3 adult tickets were purchased.
17. The following system of equations has the solution (4, 7).  
      
    What are the values of and ?

## 6.2 The Elimination Method for Three Equations With Three Unknowns

**Key Ideas**

With three equations and three unknowns, the solution can be an ordered triple (x, y, z). The process of solving a system of equations with three unknowns requires three different elimination steps.

A system of three equations with three unknowns looks like this.

**Step 1:**

Decide which variable to eliminate first.

The first step in solving a system of three equations with three unknowns is to examine the coefficients of each of the variables to see if the variables for any coefficient are matching numbers or have numbers that are multiples of the other coefficients. Any variable can be eliminated. However, it is easier to eliminate a variable when the coefficients contain multiples of other coefficients.

In this example, the -variables have the coefficients -2, +2, and -1. The will be the variable we try to eliminate.

**Step 2:**

Combine the first and second equation to eliminate a variable.

Since -2 is the opposite of +2, this is like Case 1 from the previous section. The can be eliminated by adding the two equations together. This step could require multiplying both sides of one or both equations by a constant before combining them.

This equation will be used in Step 4.

**Step 3:**

Combine the second and third equation (or the first and third if it is easier) to eliminate a variable.

Since the will not eliminate , both sides of the second equation must first be multiplied by +2 so the -coefficient will become -2, the opposite of the coefficient of the other , +2.

Now the equations can be combined to eliminate .

This equation, together with the one from Step 2, is used in Step 4.

**Step 4:**

Solve the resulting system of two equations with two unknowns.

Since the +6 is a multiple of the -3, multiply both sides of the first equation by +2.

Substitute into either of the two-variable equations.

**Step 5:**

Substitute the solution of the two-variable system of equations into any of the original equations to find the solution for the third variable.

Any of the original three equations can be used.

The solution is the ordered triple (5, 2, 1).

This can be checked by substituting into each of the three original equations.

**Example**

Find the solution to the system of equations.

Solution: Since the -variables have coefficients of +5, -5, and +5, the should be eliminated. Combining equations 1 and 2 gets . Combining equations 2 and 3 gets .

Multiply both sides of the second equation by -2 so the -coefficient becomes -4.

Substitute into one of the two-variable equations.

Now substitute and into one of the original equations.

The solution is (4, -3, 1).