**Barron’s Let’s Review Regents – Algebra II**

# Chapter 6: Systems of Equations

## 6.1 Two Equations with Two Unknowns

**Key Ideas**

A *system of two equations* with two unknowns is two equations each containing two variables. The solution set to a system of two equations with two unknows is usually an ordered pair that makes both equations true.

The ordered pair (3, 7) is a solution to the equation . If the first number for the and if the second number is substituted for the , the equation becomes , which is true. Other ordered pairs that are solutions to this equation are (8, 2), (5, 5), (10, 0), and (12, -2).

An example of a system of two equations with two unknowns is

The goal is to find, if one exists, and ordered pair that satisfies both equations at the same time. By examining different ordered pairs that make the first equation true and by testing to see if they also make the second equation true, it is possible through the process of guess-and-check to find that the ordered pair (7, 3) is the solution because and   
. For more complicated examples, though, a method that uses algebra is needed.

**The Substitution Method**

If one of the variables is already isolated in one of the equations, it is possible to simplify the two equations with two variables into just one equation with one variable.

Since is isolated in the first equation, the in the second equation can be replaced with .

To get the other number in the ordered pair, substitute into the first equation.

The ordered pair that satisfies both equations is (4, 10).

**Example 1**

Find the solution set of the system of equations.

*Solution*: Substitute for in the second equation.

Substitute into one of the original equations.

The solution is (6, 7).

**The Elimination Method**

If the two equations can be combined in a way that eliminates one of the variables, the remaining equation can be solved for one part of the ordered pair. Sometimes, this method requires that you multiply both sides of one or of both equations by some constant.

**Case 1:**

A coefficient of one variable is the opposite of the coefficient of the same variable in the other equation.

Examine the coefficients of the system of equations.

The coefficient of the in the first equation is 3.

The coefficient of the in the second equation is -3. Rember that . If these equations are added together, the result with be an equation with no -term.

Substitute into either original equation and solve for .

The solution to the system of equations is the ordered pair (4, 1).

**Case 2:**

A coefficient of one of the variables in one of the equations is equal to the coefficient of the same variable in the other equation.

Examine the coefficients in the system of equations.

The coefficients of the -terms in both equations are equal to +4.

It is permitted to multiply both sides of an equation by the same constant. For this situation, multiply both sides of either equation by -1. Below is what happens if both sides of the second equation are multiplied by -1.

Substitute 3 for in either of the original equations.

The solution set is (5, 3).

**Case 3:**

A coefficient of one of the variables in one of the equations is a multiple of the same variable in the other equation.

Examine the coefficients in the system of equations.

The coefficient of the in the first equation, 6, is a multiple of the coefficient of the in the second equation, 3. For the equation that has the smaller coefficient, multiply both sides of the equation by the number that would make the coefficient the opposite of the one from the other equation. For this example, multiply both sides of the second equation by -2.

Now substitute into one of the original equations.

The solution is (8, -2).

**Case 4:**

No coefficient for either variable in either equation is a multiple of the coefficient of the same variable in the other equation.

Examine the coefficients of the system of equations.

For the -terms, the 5 is not a multiple of 3. For the   
-terms, the -6 is not a multiple of the 4. When this happens, both equations must be changed.

You can choose whether you want to change the equations so that either the -variables will be eliminated or the -variables will be eliminated.

To eliminate the -variables, find the least common multiple of the two coefficients. In this case, the least common multiple of 4 and 6 is 12. The goal is to convert one of the y-coefficients into a +12 and the other into a -12. This can be done by multiplying both sides of the first equation by +3 and both sides of the second equation by +2.

Substitute 10 for into either of the original equations.

The solution is (10, 3).

**Example 2**

Solve the system of equations by eliminating the -terms.

Solution: The least common multiple of the   
-coefficients is 12. Multiply both sides of the first equation by +4 and both sides of the second equation by -3.

Substitute into one of the original equations.

The solution is (6, -2).

**Math Facts**

Sometimes when solving a system of equations with the elimination, both variables get eliminated and the resulting equation has just numbers. If the two numbers of the resulting equation are not equal, there is no solution to the original system. If the two numbers of the resulting equation are equal, there are an infinite number of solutions to the equation.

**Example 3**

How many solutions does this system of equations have?

(1) No solutions  
(2) One solution  
(3) Two solutions  
(4) More than two solutions

Solution: When the two equations are added together, the result is , which is never true. There are no solutions to the original system of equations. Choice (1) is correct.

**Linear-Quadratic Systems of Equations**

If one of the equations has an exponent of 2, the system can be solved with the substitution method. The resulting equation will be a quadratic equation with up to two solutions.

**Example 4**

What ordered pair(s) satisfies the system?

*Solution*:

For . For . So the two solutions are (-1, 0) and (2, 3).